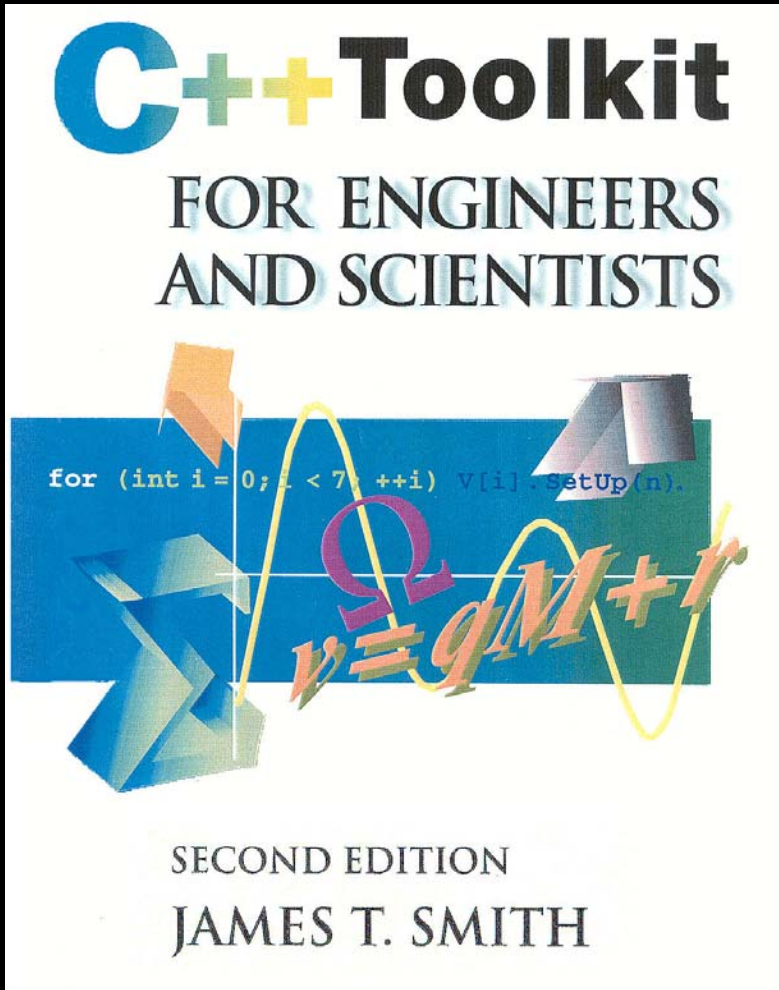
Claremont History and Philosophy of Mathematics Seminar
Harvey Mudd College

1 April 2019

Mario Pieri, Overloading, and Information Hiding in 1906

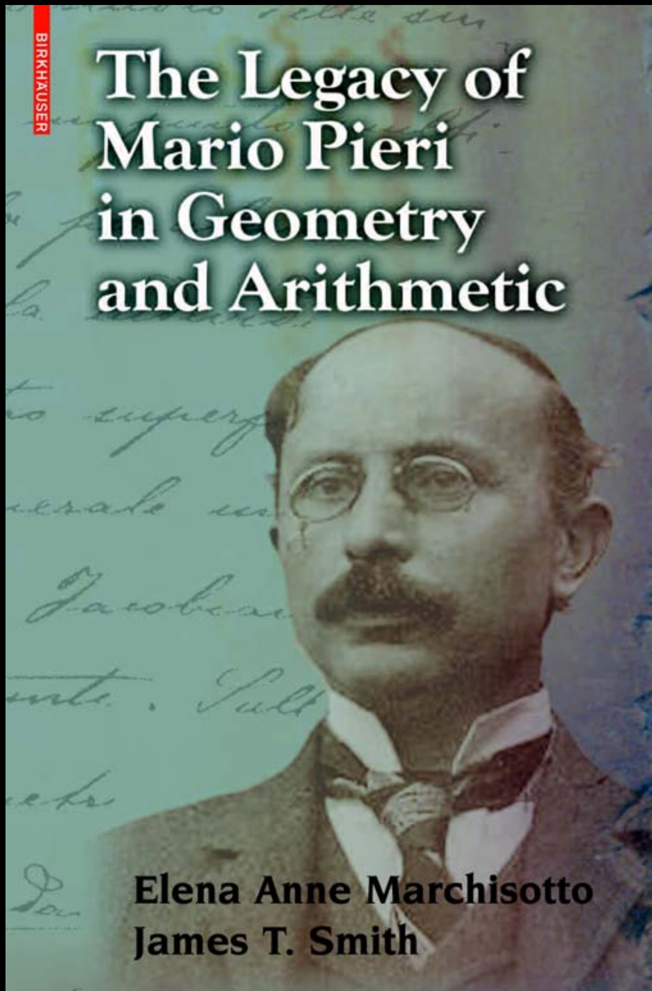
James T. Smith, Professor Emeritus
San Francisco State University



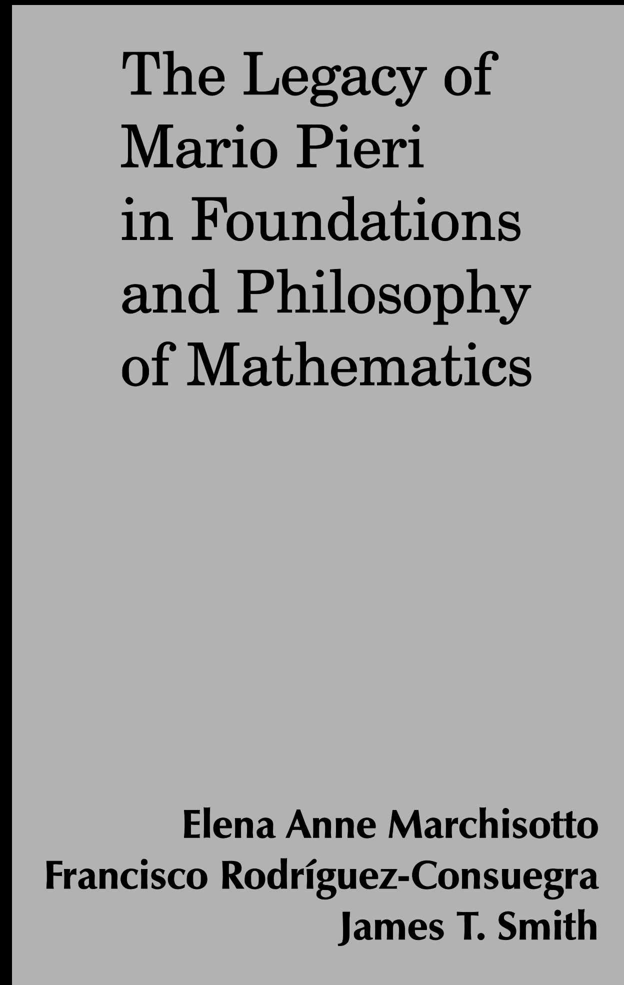
Springer, 1999

My talk *ends* on this topic.

But it *begins* with work on the next slide.

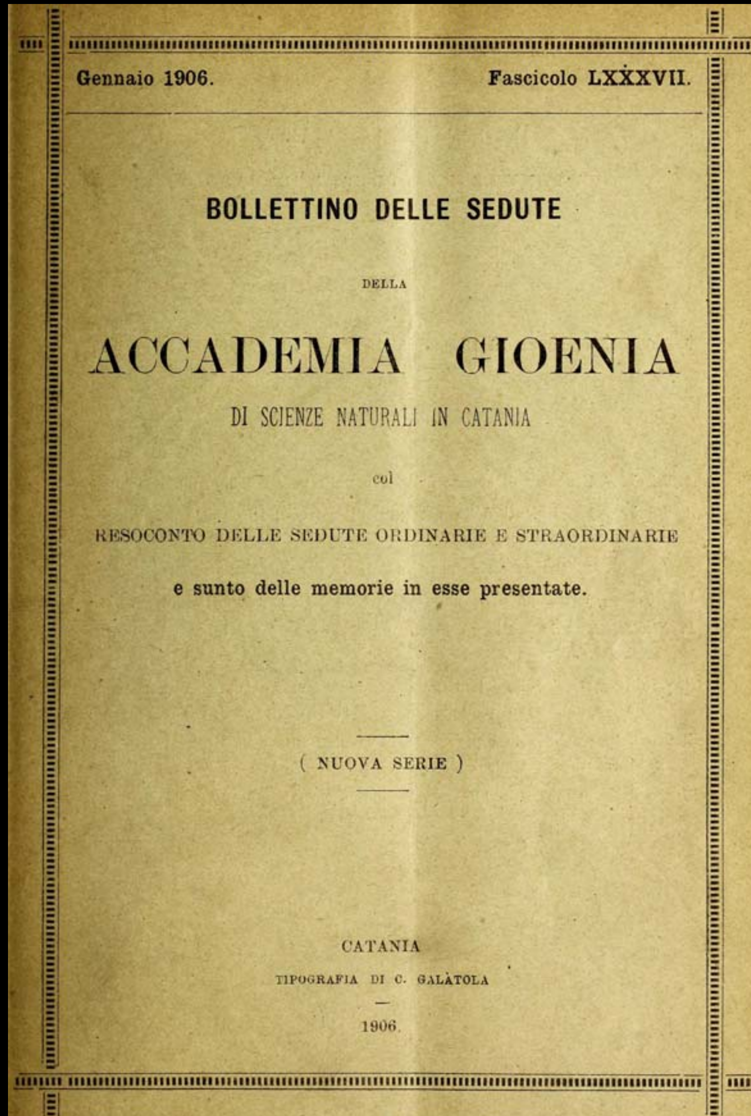


Birkhäuser, 2007



In preparation

- Mario Pieri (1860–1913)
- Algebraic & enumerative geometry
- Foundations and philosophy of mathematics
- Obscure paper anticipates developments 50+ years later.
- 1906. *On an Arithmetical Definition of the Irrationals*.
- Giuseppe Gioeni D’Angiò (1747–1822), vulcanologist.



Gennaio 1906. Fascicolo LXXXVII.

BOLLETTINO DELLE SEDUTE

DELLA

ACCADEMIA GIOENIA

DI SCIENZE NATURALI IN CATANIA

col

RESOCONTO DELLE SEDUTE ORDINARIE E STRAORDINARIE

e sunto delle memorie in esse presentate.

(NUOVA SERIE)

CATANIA

TIPOGRAFIA DI C. GALÀTOLA

1906.

Introducing Real Numbers

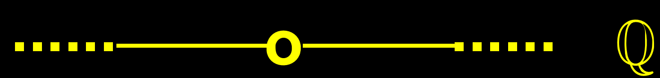
- **Weierstrass, 1878: equivalence classes of formal sums of quotients of whole numbers**

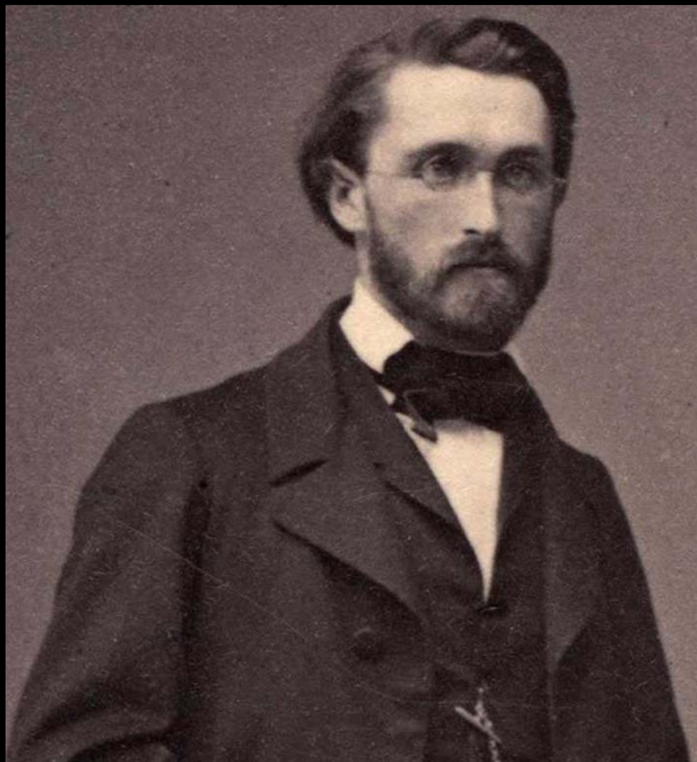
$$\sum_{k=0}^{\infty} \frac{n_k}{d_k} \qquad d_k > 0$$

- **Cantor, 1872: equivalence classes of "Cauchy" sequences $\langle x_k \rangle$ of rationals**

$$(\forall \varepsilon > 0)(\exists k)(\forall m, n \geq k)$$
$$|x_m - x_n| < \varepsilon$$

- **Dedekind, 1872: cuts in rationals**



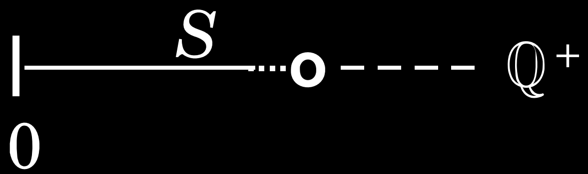


Richard Dedekind
(1831–1916)

Moritz Pasch
(1843–1930)

Moritz Pasch, 1882:

- $\mathscr{S}$ = family of nonempty



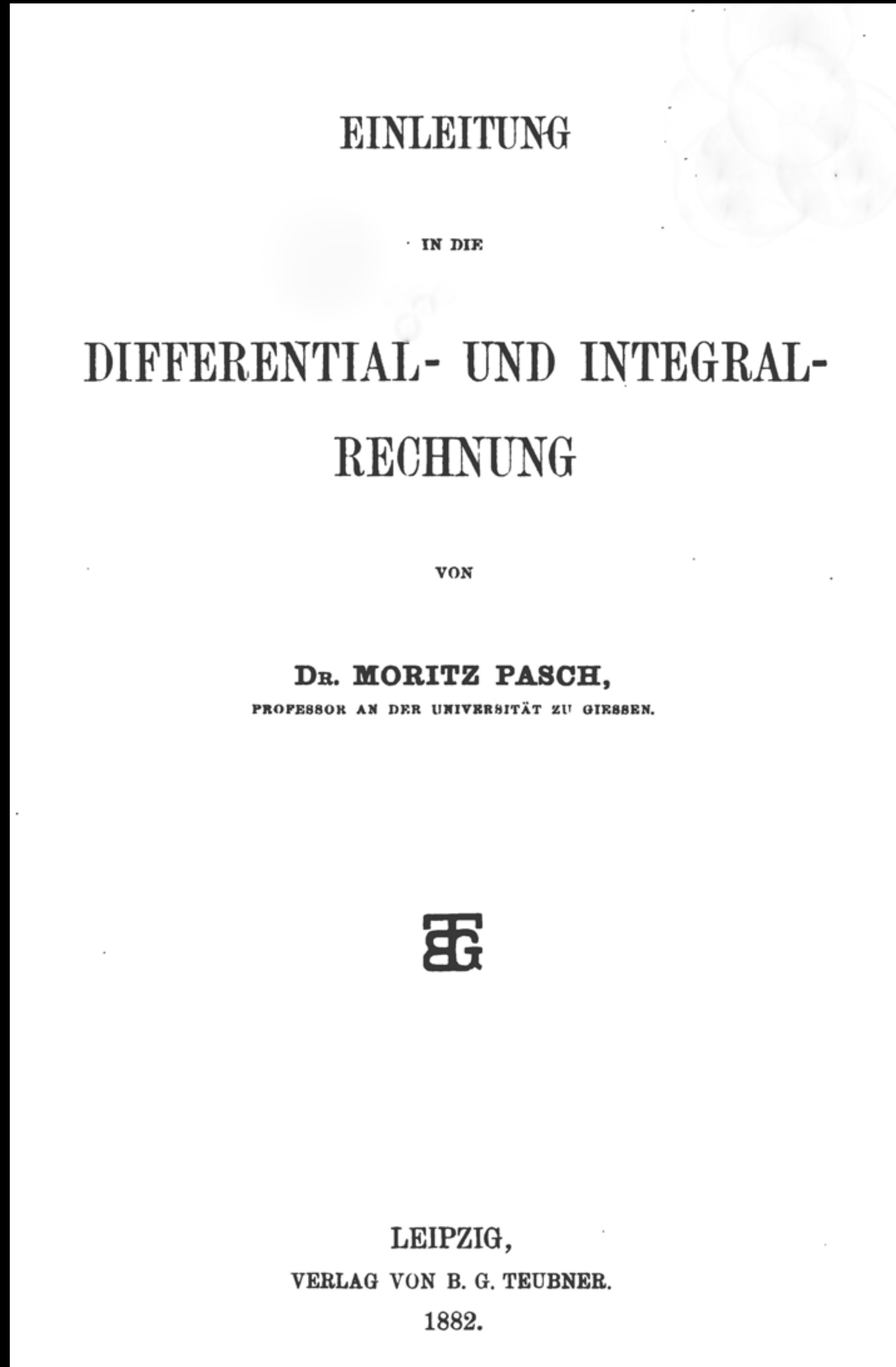
$$p < q \in S \;\Rightarrow\; p \in S$$
$$p \in S \;\Rightarrow\; (\exists q \in S)\, p < q$$

- For $S, T \in \mathscr{S}$ define

$$S < T \;\Leftrightarrow\; S \subsetneq T$$

$$S + T \;=\; \{\, s + t \;:\; s \in S \;\&\; t \in T \,\}$$

etc.



EINLEITUNG

IN DIE

DIFFERENTIAL- UND INTEGRAL-RECHNUNG

VON

DR. MORITZ PASCH,

PROFESSOR AN DER UNIVERSITÄT ZU GIESSEN.

LEIPZIG,

VERLAG VON B. G. TEUBNER.

1882.

Pasch 1882, 11:

Rational segments

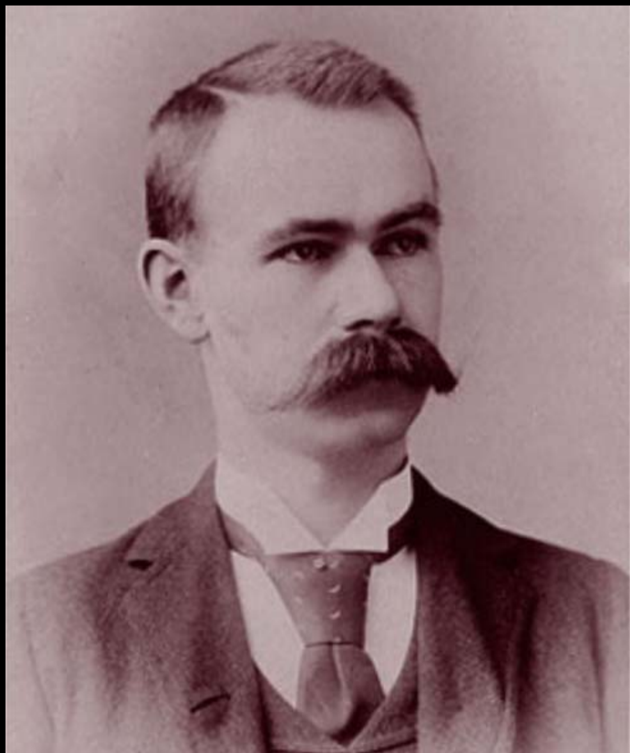
$$S_q = \{p \in \mathbb{Q}^+ : p < q\}$$

have the same properties as the rationals $q \in \mathbb{Q}^+$.

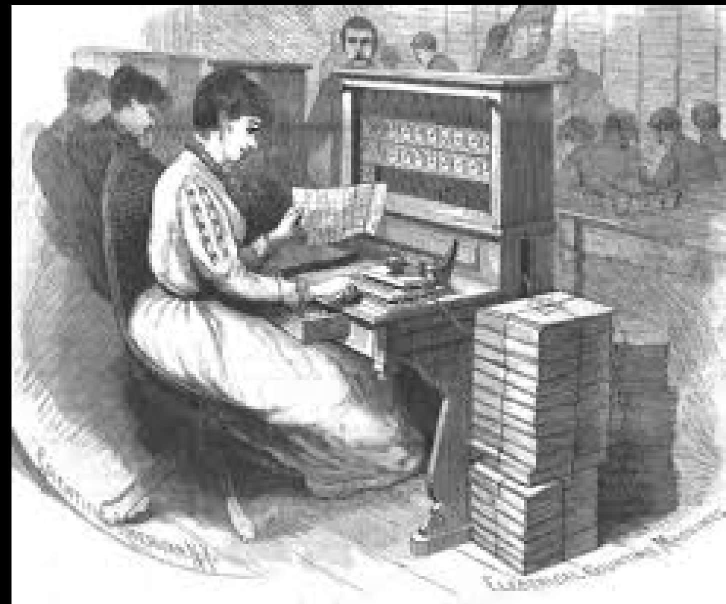
The other segments have the properties we want for the irrationals.

From now on *number* will mean *segment*. Some numbers are rational and the others, *irrational*.

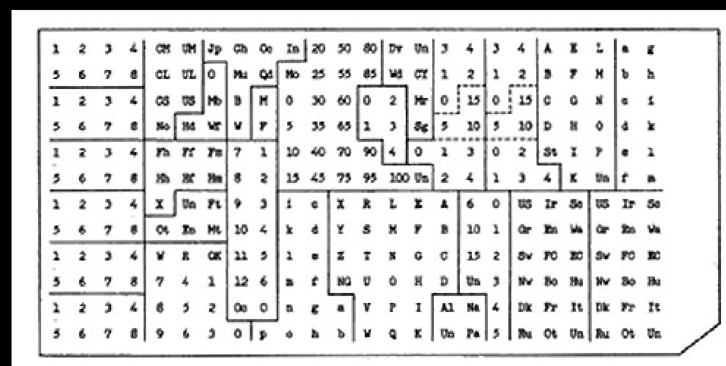
Meanwhile,
across
the sea ...



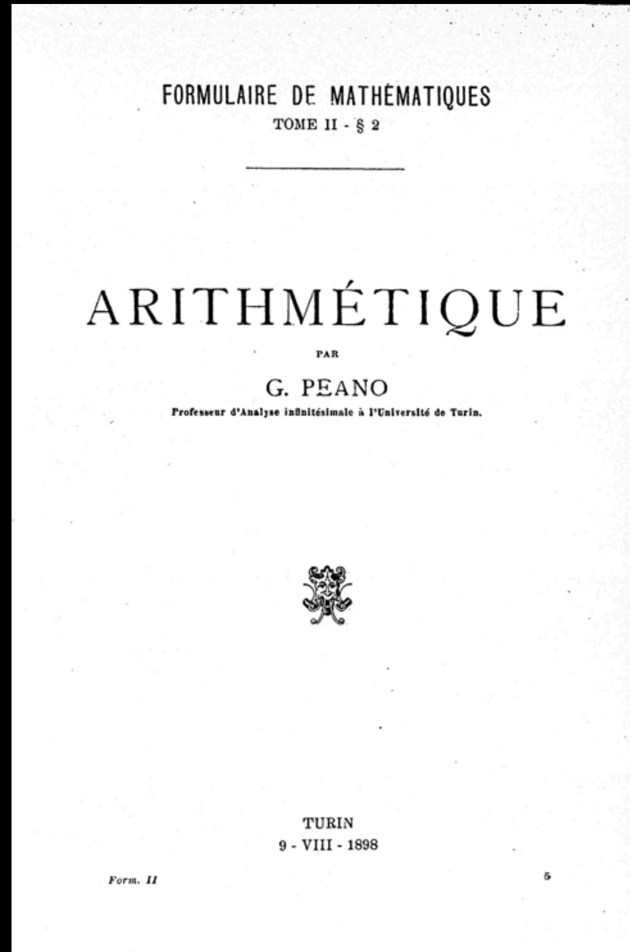
Herman Hollerith
(1860–1929)



Tabulating the 1890 U. S. Census



Giuseppe Peano
(1858–1932)



FORMULAIRE DE MATHÉMATIQUES
TOME II - § 2

ARITHMÉTIQUE

PAR

G. PEANO

Professeur d'Analyse infinitésimale à l'Université de Turin.

TURIN
9 - VIII - 1898

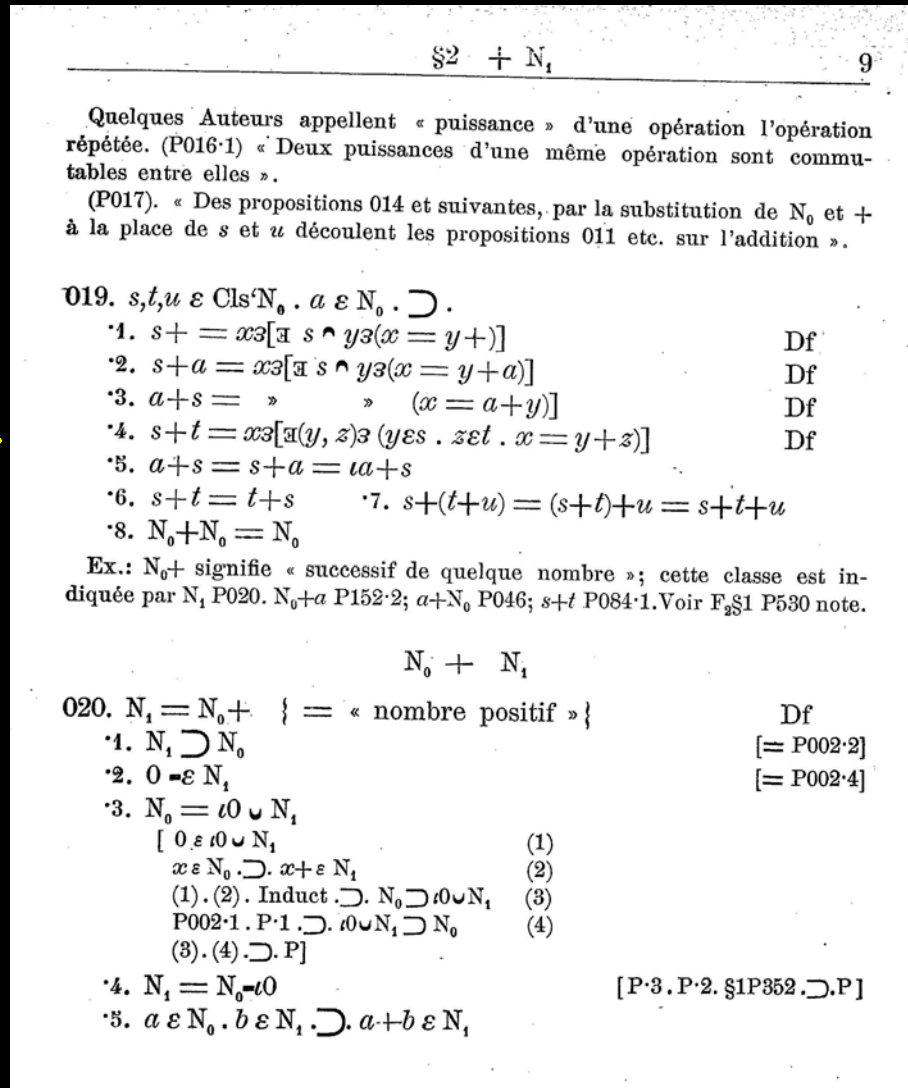
Form. II 5

1898

- **Peano was trying to put *all* mathematics into a *single* system, his *Formulary*.**

- **Definition of $s+t$:**

(1898, page 9)



§2 $+ \; N_1$ 9

Quelques Auteurs appellent « puissance » d'une opération l'opération répétée. (P016·1) « Deux puissances d'une même opération sont commutables entre elles ».

(P017). « Des propositions 014 et suivantes, par la substitution de N_0 et $+$ à la place de s et u découlent les propositions 011 etc. sur l'addition ».

019. $s,t,u \; \varepsilon \; \mathrm{Cls}`N_0 \,.\, a \; \varepsilon \; N_0 \,.\supset .$

·1. $s+ \; = x \ni [\exists \; s \cap y \ni (x = y+)]$ Df

·2. $s+a = x \ni [\exists \; s \cap y \ni (x = y+a)]$ Df

·3. $a+s =$ » » $(x = a+y)]$ Df

·4. $s+t = x \ni [\exists (y, z) \ni (y \varepsilon s \,.\, z \varepsilon t \,.\, x = y+z)]$ Df

·5. $a+s = s+a = \iota a + s$

·6. $s+t = t+s$ ·7. $s+(t+u) = (s+t)+u = s+t+u$

·8. $N_0 + N_0 = N_0$

Ex.: N_0+ signifie « successif de quelque nombre »; cette classe est indiquée par N_1 P020. N_0+a P152·2; $a+N_0$ P046; $s+t$ P084·1. Voir F_2§1 P530 note.

$N_0 \; + \; N_1$

020. $N_1 = N_0 +$ { = « nombre positif » } Df

·1. $N_1 \supset N_0$ [= P002·2]

·2. $0 \, -\varepsilon \, N_1$ [= P002·4]

·3. $N_0 = \iota 0 \cup N_1$

[$0 \; \varepsilon \; \iota 0 \cup N_1$ (1)

$x \; \varepsilon \; N_0 \,.\supset.\, x+ \; \varepsilon \; N_1$ (2)

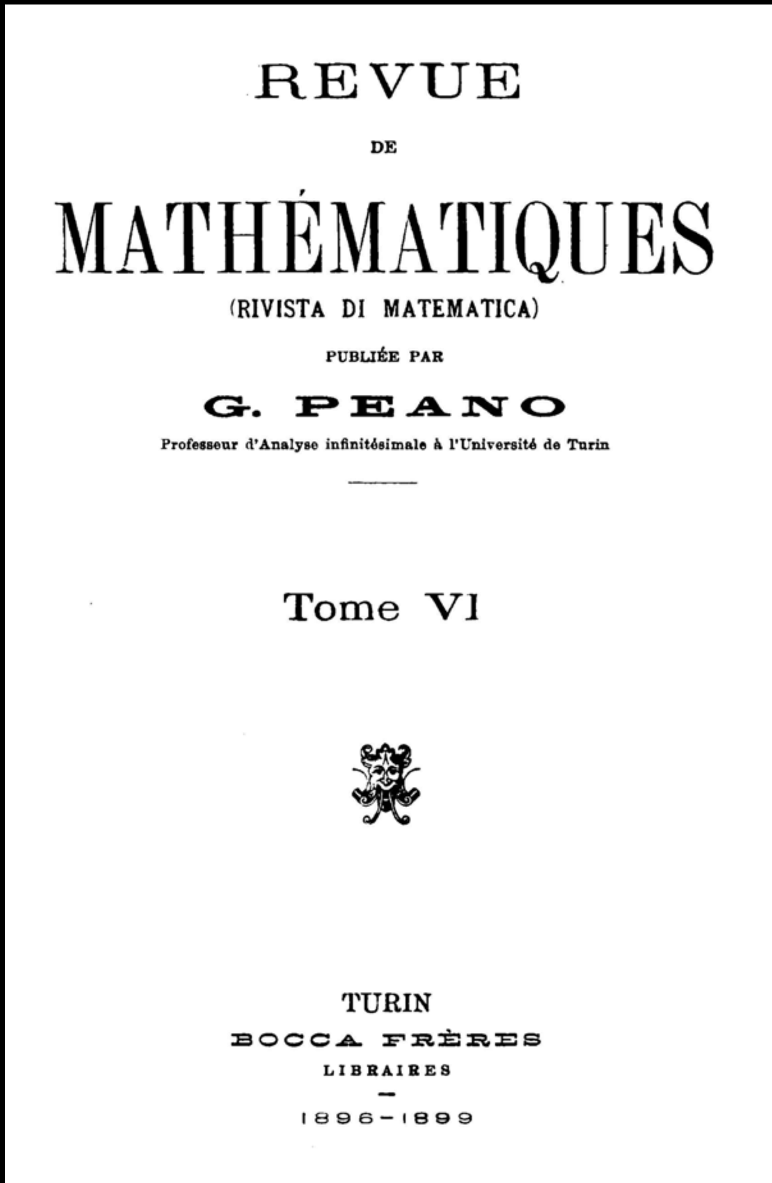
(1) . (2) . Induct $.\supset.\; N_0 \supset \iota 0 \cup N_1$ (3)

P002·1 . P·1 $.\supset.\; \iota 0 \cup N_1 \supset N_0$ (4)

(3) . (4) $.\supset.$ P]

·4. $N_1 = N_0 - \iota 0$ [P·3 . P·2. §1P352 $.\supset.$P]

·5. $a \; \varepsilon \; N_0 \,.\, b \; \varepsilon \; N_1 \,.\supset.\; a+b \; \varepsilon \; N_1$


REVUE

DE

MATHÉMATIQUES

(RIVISTA DI MATEMATICA)

PUBLIÉE PAR

G. PEANO

Professeur d'Analyse infinitésimale à l'Université de Turin

Tome VI

TURIN

BOCCA FRÈRES

LIBRAIRES

1896–1899

Peano 1899. Sui numeri irrazionali. *Rivista di matematica* 6: 126–140.

- By his rules, $S \in \mathcal{S} \Rightarrow$

$$S^2 = \{s^2 : s \in S\}.$$

- If $S = \{s \in \mathbb{Q}^+ : s < 3\}$
  then $4 \in S^2$.
  But $2 < 4$ and $2 \notin S^2$,
  so $S^2 \notin \mathcal{S}$.
- Instead, we want

$$S \times S = \{s \times t \ : \ s, t \in S\}.$$

- **We'd need a new operation:**

$$S \uparrow 2 \,.$$

Peano didn't like that:

"It is possible, speaking always of segments, to construct a complete theory of the irrationals; but the formulas, if it is desired to exclude all danger of ambiguity, would be presented in a form altogether different from that in use today in algebra. While admitting that this form is appropriate for the civilizations in which we live,…it is altogether necessary that our notations agree with those used by everyone."

(1899, page 9)

Russell, Bertrand. 1903. *Principles of Mathematics*, 286.

"…there is no logical ground for distinguishing segments of rationals from real numbers. If they are to be distinguished, it must be in virtue…of some wholly new axiom…The above theory, on the contrary, requires no new axiom:…an irrational ***actually is*** a segment of rationals which does not have a limit."

Bertrand Russell
(1872–1970)

Some Italian Cities

● **Public universities in Pieri's time**

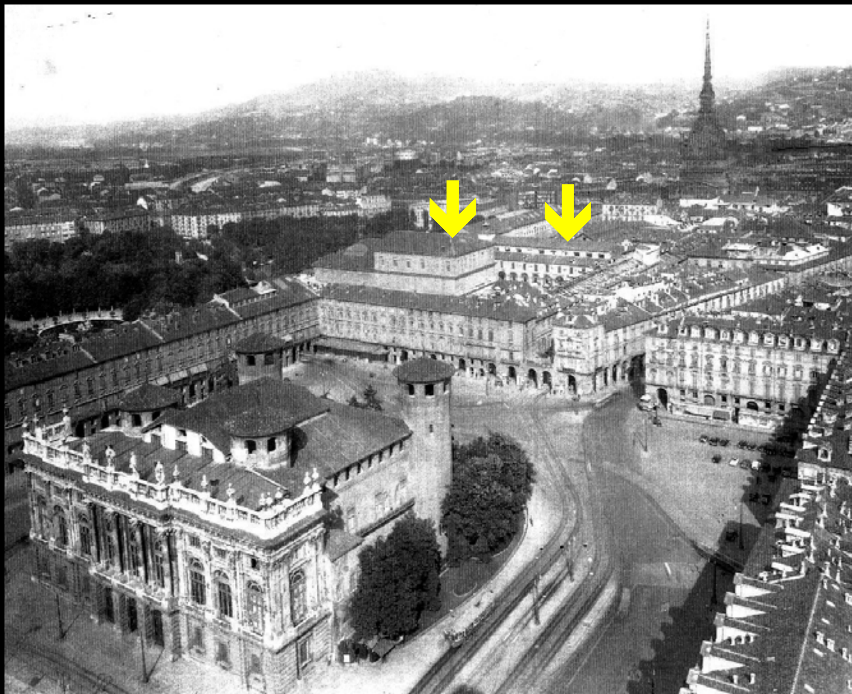
○ **Other cities**

Mario Pieri (1860–1913)

- Born in Lucca (population 22,800) in Piazza San Giusto

From Pieri’s Front Steps

- Father a lawyer
- Schooled in Lucca, Bologna
- 1884 laureate, Pisa (20 km): algebraic and differential geometry
- 1886, professor of projective, descriptive geometry, Regia Accademia Militare, Turin
- 1888, assistant, Univ. Turin
- Worked with Corrado Segre, Giuseppe Peano



← *Mole Antonelliana*

← *Corso San Maurizio*

← *Royal Theater, Military Academy*

← *University of Turin*

← *Via Verdi (Via della Zecca)*

← *Castello*

Turin, Piazza Castello, 1934

(1886 population ≈ 290,000)

Mario Pieri around 1895

1898 Axiomatization of Real Projective Geometry

THE PRINCIPLES

OF THE

GEOMETRY OF POSITION

COMPOSED INTO A DEDUCTIVE LOGICAL SYSTEM

MEMOIR

BY

MARIO PIERI

LIBERO DOCENTE AT THE UNIVERSITY OF TURIN

Approved at the session of 19 December 1897.

INTRODUCTION

> ...*Foundation* of these first elements is a task certainly not less difficult than the further development of the most complicated theorems. That pushes into the depths, this into the heights: and the depths and heights are equally unbounded and dark...
> ...surely, however, the interest in these is rather great, because the inherent property of mathematics, and with it geometry, to be one of the few parts of human knowledge that enjoy *complete* certainty and truth, will become doubtful as soon as the theorems and concepts on which it is based become shaky.
> August CRELLE, *On the Theory of the Plane*[4]

Projective geometry has been regarded for a long time as a simple continuation of elementary geometry; and by most is still preferred, to establish its principles with successive extensions of the concepts that govern elementary geometry, derived from observation of the external world and fully conforming to the idea that they are acquired through experimental induction from certain qualities of physical objects and facts. In this way geometry, also preserving in its methods that deductive character which stems from the most remote antiquity, is always presented as an aspect of the *physics of extension*, rather than as taking a position alongside Analysis among the purely mathematical disciplines.

[4] Crelle 1853, 20, 17. [The italics are Crelle's. They are different in Pieri 1898c.]

- In 1900, finally, Pieri was appointed professor, in Catania
- (1900 population ≈ 191,000)

- Pieri married Angiolina Anastasio In 1901.
- They lived in Via Gesuiti.

Recall Peano 1899, page 9:

"It is possible, speaking always of segments, to construct a complete theory of the irrationals; but the formulas, if it is desired to exclude all danger of ambiguity, would be presented in a form altogether different from that in use today in algebra. While admitting that this form is appropriate for the civilizations in which we live,…it is altogether necessary that our notations agree with those used by everyone."

Pieri, Mario. 1906. Sopra una definizione aritmetica degli irrazionali. *Bollettino delle sedute della Accademia Gioenia di Scienze Naturali in Catania* 87: 14–22.

"This [fault] deserves to be acknowledged and possibly removed….It suffices to identify each irrational (or real) number not just with such a segment…but with the same class *considered as…an individual*…. [This new] notion might be a most useful tool…."

Pieri's Suggestion

New logical operator: I

New logical axioms:

- S **is a class of individuals (non-classes)**
  $\Rightarrow$ IS **is an individual &** $IS \notin S$**.**
- S, T **are classes of individuals** $\Rightarrow$ $(IS = IT \iff S = T)$**.**

New definitions:

- $\mathbb{R}^+ = \{IS : S \in \mathcal{S}\}$
- **for** $x, y \in \mathbb{R}^+$,
  $x < y \iff (\exists S, T \in \mathcal{S})(x = IS \ \& \ y = IT \ \& \ S \subsetneq T)$
- **etc.**

Lack of impact

- **Pieri 1906 attracted *virtually no attention*,**
- **even though it could be extended to yield various *types* of individuals.**
- **Mathematics progressed in another way: today's familiar informal modular approach.**
- **Peano's project lapsed.**
- **……..**

Fifty years later

UNIVAC I

Sold to the Census

Peano & Pieri	*Object-Oriented Programming*

• Grand system for all math
• For use in instruction...
• Ideography was a tool.
• Familiar notation for high-level operators,
  • with arguments of varying but analogous types,
  • without colliding with lower-level usage...
• Hide lower-level detail !
	• Large-scale electronic manipulation of data of different but analogous types, using different algorithms for analogous operations...
	• Overloading enhances reliability by making programming more intuitive.
	• Information hiding fosters portability and prevents disruption of low-level computations by higher-level software errors.

- The aim of the C++ class concept ... is to provide the programmer with a tool for creating new types that can be used as conveniently as the built-in types.
- The fundamental idea in defining a new type is to separate the incidental details of the implementation ... from the properties essential to the correct use of it.
- There are several benefits to be obtained from restricting access to a data structure to an explicitly declared list of functions.

Bjarne Stroustrup
(1950–)

The C++ Programming Language. 2nd edition. Addison-Wesley, 1991. 143–146.

I hope some historians will pursue this parallelism more deeply!

Thank you for your interest!

James T. Smith
Professor Emeritus
San Francisco State University

Mario Pieri, Overloading and Information Hiding in 1906

James T. Smith

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———. 1906. Sopra una definizione aritmetica degli irrazionali. *Bollettino delle sedute della Accademia Gioenia di Scienze Naturali in Catania* 87: 14–22.

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